The Landau gauge lattice QCD simulation and the gluon propagator

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The gluon propagator in the Landau gauge lattice QCD simulation is measured. The data suggests the confinement mechanism of the Gribov-Zwanziger theory.

1. Introduction

In 1978 Gribov showed that the fixing of the divergence of the gauge field in non-Abelian gauge theory does not fix its gauge[1]. The restriction of the gauge field such that its Faddeev-Popov determinant is positive (Gribov region) is necessary yet insufficient for excluding the ambiguity.

The restriction to the fundamental modular region[2,4] cancels the infrared singularity of the perturbation theory and makes the gluon to have the complex mass. The transverse gluon propagator in the continuum theory is defined as

$$
\begin{aligned}
D_{\mu\nu}(k) &= \frac{1}{n}\sum_{x=\mathbf{x},t} e^{-ikx} Tr\langle A_\mu(x)A_\nu(0)\rangle \\
&= G(k^2)(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}) \ .
\end{aligned} \tag{1}
$$

where n is the dimension of the adjoint representation of SU(N). Schematically

$$
G(k^2) = \frac{k^2}{k^4 + \kappa^4} = \frac{1}{2}(\frac{1}{k^2 + i\kappa^2} + \frac{1}{k^2 - i\kappa^2}). \tag{2}
$$

It was shown[2] that in the lattice simulation of the non-Abelian gauge theory, the Landau gauge fixing and the restriction of the gauge field yields the gluon propagator similar to that of Gribov.

2. The global gauge fixing and the gluon propagator

We performed quenched ($4^3 \times 8$ and $8^3 \times 16$) and unquenched ($4^3 \times 8$) lattice QCD simulation in the Landau gauge whose algorithm is presented in[3,4]. The number of configuration is 100 for the three cases.

After the Landau gauge fixing with an accuracy $\text{Max}|\partial_\mu A_\mu| < 10^{-4}$ which is roughly equivalent to $(\partial_\mu A_\mu)^2 < 10^{-10}$, we fix the global gauge of the field $A_\mu(x)$ such that $A = \sum_{\mu x a} A_\mu^a(x)\lambda^a$ of a sample is diagonalized by an SU(3) matrix g : $g^\dagger A g = diag(g_1, g_2, g_3), (g_1 \geq g_2 \geq g_3)$.

We obtain the new data sets $A^g_\mu(x) = g^\dagger A_\mu(x)g$ and perform the Fourier transform $A^{ga}_\mu(n_k) = \frac{1}{\sqrt{N_x^3 N_t}}\sum_x e^{-ik\cdot(x+e_\mu/2)}A^{ga}_\mu(x)$.

Using the projection operator on a lattice in the momentum space

$$
P_{\mu\nu}(n_k) = \delta_{\mu\nu} - \frac{c_\mu(n_k)c_\nu(n_k)}{c(n_k)^2} \tag{3}
$$

where $c_\mu(n_k) = 2\sin\frac{\hat{k}_\mu}{2}$, $c(n_k)^2 = \sum_{\mu=1}^4 c_\mu(n_k)^2$ and $\hat{k}_\mu = \frac{2\pi n_k}{N_\mu}$ ($n_k = 0, \cdots, N_\mu - 1$) is the available momentum on the lattice in the $\mu-$direction, the gluon propagator is calculated from

$$
\begin{aligned}
D(\hat{k}) &= \sum_{\mu\nu a} P_{\mu\nu}(n_k)(\langle A^{ga}{}_\mu(n_k)A^{ga}{}_\nu(n_k)^*\rangle \\
&\quad - \langle A^{ga}{}_\mu(n_k)\rangle\langle A^{ga}{}_\nu(n_k)^*\rangle) \tag{4}
\end{aligned}
$$

where $\langle A^{ga}{}_\mu(n_k)\rangle$ is the average over samples of the globally gauge fixed field $A^{ga}_\mu(n_k)$.

We define the gluon propagator as a function of $\hat{k} = \sqrt{\sum_\mu \hat{k}_\mu^2}$ where $\hat{k}_\mu = \mathrm{Min}[n_k, N_\mu - n_k]\frac{2\pi}{N_\mu}$ and not a function of $k = \sqrt{\sum_\mu (2\sin\frac{\hat{k}_\mu}{2})^2}$.

In the case of $8^3 \times 16$ lattice there appear 4096 independent four-momentum $\hat{k}_\mu$. For a fixed $\hat{k}$, the data of $D(\hat{k})$ from $0 \leq n_k < \frac{N_\mu}{2}$ and $\frac{N_\mu}{2} \leq n'_k < N_\mu$ which satisfies $N_\mu - n'_k = n_k$ are plotted at the same abscissa and in the medium momentum region the difference of the data at n'_k and n_k makes the plot of $D(\hat{k})$ scattered. This effect could be attributed to the finite size effect and in larger lattice the effect becomes smaller[5].

We adopt the data selection prescription similar to that of [5], i.e. select data such that $|n_{ki} - n_{kj}| \leq 1$ where i and j run from 1 to 4, or choose the momentum $\hat{k}$ such that it is almost diagonal in the $N_x^3 \frac{N_t}{2}$ lattice space. In fig. 1 we present the result of $D(\hat{k})$. The data clearly shows the infrared suppression of the gluon propagator. The suppression remains even without the subtraction of $\langle A^{ga}{}_\mu(n_k)\rangle\langle A^{ga}{}_\nu(n_k)^*\rangle$ term.

We performed the analysis also for $\hat{k}$ along the spacial axis[6]. The results of $8^3 \times 16$ lattice suggests that the rotational symmetry is not so good. Although in the quenched $4^3 \times 8$ lattice the gluon propagator did not show the significant suppression and remained finite, in the unquenched $4^3 \times 8$ lattice similar suppression was observed[6]. In the SU(2) case similar effect is also observed[7].

3. Parametrization of the gluon propagator

In order to get insights on the complex mass of the confined gluon, we performed an analysis of the Fourier transform of the correlation function

$$D_T(\mathbf{k}, t) = \frac{1}{2n}\sum_{jTx} e^{i\mathbf{k}\cdot\mathbf{x}} Tr\langle A_j(\mathbf{x}, T+t)A_j(\mathbf{0}, T)\rangle_c \quad (5)$$

where the suffix c indicates that the connected part is taken after the global gauge fixing and

$$\tilde{A}_\mu^a(\mathbf{k}, T) = \frac{1}{N_x^3}\sum_x A_\mu^a(\mathbf{x}, T)e^{-i\mathbf{k}\cdot\mathbf{x}}. \quad (6)$$

In the Stingl's factorizing-denominator rational approximants (FDRA) method[8], the transverse
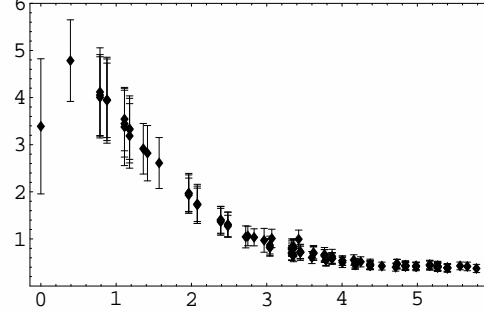

Figure 1. The gluon propagator $D(\hat{k})$ of quenched $\beta = 5, 8^3 \times 16$ lattice.

gluon propagator is expressed as

$$D_T(p^2)^{[r=3]} = \frac{\rho}{p^2 + u_+\Lambda^2} + \frac{\tau}{p^2 + v_+\Lambda^2} + c.c. \quad (7)$$

which is compared with its lattice Fourier transform

$$
\begin{aligned}
G_T(\mathbf{k}, t)_c &= Re[(a_0 + ia_1)\cosh((a_2 + ia_3)(t-4)) \\
&+ (a_4 + ia_5)\cosh((a_6 + ia_7)(t-4))] \\
&= Re[\frac{\rho a}{\sinh(\hat{M}_1 a)\sinh(\frac{N_T}{2}\hat{M}_1 a)}\cosh(\hat{M}_1 a(t-4)) \\
&+ \frac{\tau a}{\sinh(\hat{M}_2 a)\sinh(\frac{N_T}{2}\hat{M}_2 a)}\cosh(\hat{M}_2 a(t-4))] \quad (8)
\end{aligned}
$$

where the gluon masses are defined as $\hat{M}_1 a = a_2 + ia_3, \hat{M}_2 a = a_6 + ia_7$ and

$$\rho a = (a_0 + ia_1)\sinh(a_2 + ia_3)\sinh(N_T(a_2 + ia_3)/2), (9)$$

$$\tau a = (a_4 + ia_5)\sinh(a_6 + ia_7)\sinh(N_T(a_6 + ia_7)/2), (10)$$

$$u_+\Lambda^2 = 4\sinh^2((a_2 + ia_3)/2), \quad (11)$$

$$v_+\Lambda^2 = 4\sinh^2((a_6 + ia_7)/2). \quad (12)$$

The gluon propagator $D(p^2)$ derived in the FDRA method also shows infrared suppression. We observe that as the lattice size becomes large, the position of zero of $D(p^2)$ approaches to $p^2 = 0$ and that the peak near $p^2 = 0$ becomes sharp.
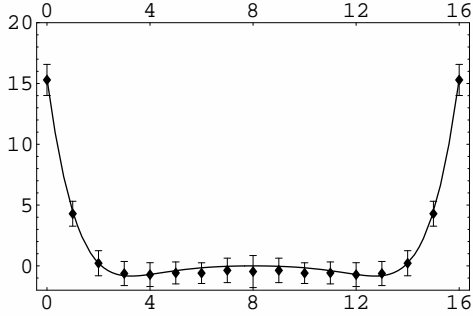
4. Discussion and outlook

We performed the Langevin simulation of the quenched and unquenched lattice QCD in the Landau gauge, using the natural definition of the gauge field.

The position of the zero of the Fourier transform of the connected part of the gluon propagator $D_T(p^2)$ approaches $p^2 = 0$ as the lattice size increases, which suggests the possibility of the realization of the confinement mechanism of the Gribov-Zwanziger theory.

Our preliminary fitting results indicate that real part of the effective mass of the gluon is about $1.4a^{-1}$ for $\beta = 4, 4^3 \times 8$ and about $0.75a^{-1}$ for $\beta = 5, 8^3 \times 16$. The renormalization and the anomalous dimension for the effective mass of the gluon is under investigation.

Figure 2. The gluon propagator $G_T(\mathbf{0}, t)$ of quenched $\beta = 5, 8^3 \times 16$ lattice.
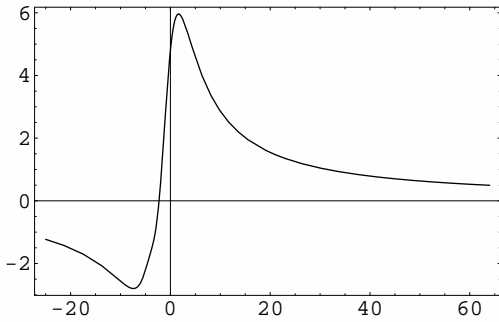
Figure 3. The lattice Fourier transformed gluon propagator $D(p^2)$.

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